The Proof Is in the Putting

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The Proof Is in the Putting

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As the saying goes, "You drive for show and putt for dough." You would think that physicists, with their superior knowledge of vectors, would make excellent putters. However, this often proves not to be the case.

In an attempt to better understand how to "read the break" on a putt, my students and I conducted a series of experiments with a spring-loaded putting machine that applies an impulse to the equator of a golf ball in roughly the same manner as a conventional putter. In this note, experimental data are compared with a simple theory of putting that excludes such things as: irregularities in the putting surface (e.g., spike marks), complex contours, dew on the grass, wind resistance, and possible ball hopping (i.e., momentarily losing contact with the green). In addition to these factors, this analysis ignores what golfers call the "grain" of the grass, or what we, in the plain language of physics, might call "textural anisotropy." In other words, this is a putting green that exists only in the mind of a physicist.

To understand how the ball will roll over the green, once struck, it is necessary to know both the slope and the frictional characteristics of the green. In order to learn how to properly model the frictional force between the ball and the putting green, we took measurements on flat and level surfaces for both a putting green and a living-room carpet. A battery-powered photogate was positioned along the track of the ball at various distances. It was then possible to compute the velocity of the ball at various positions along its path. The results of these measurements are shown in Figs. 1 and 2.

If a golf ball is struck by a putter so as to produce no initial spin,[1] then it will skid for a distance x_t until the frictional torque increases its angular velocity ω and it finally achieves the pure rolling-without-skidding condition:[2]

$$\omega = \frac{V_{cm}}{R}. \tag{1}$$

This transition from skidding to rolling is more easily observed at a bowling alley. Sometimes a bowling ball is released with no initial spin and sometimes it's even released with backspin.

To determine x_t, the distance over which the ball skids before making the transition to pure rolling, some assumptions must be made. We start by assuming that the golf ball of mass m and radius R can be treated as a uniform solid sphere.[3] For simplicity, we further assume that the coefficient of rolling friction μ_r is much smaller than the coefficient of kinetic friction μ_k and that the ball is launched with no initial angular velocity. V_o is the initial velocity, and V_f is the velocity of the ball when it has first stopped skidding and is just beginning to undergo pure rolling. Then, applying the impulse equations for both linear and

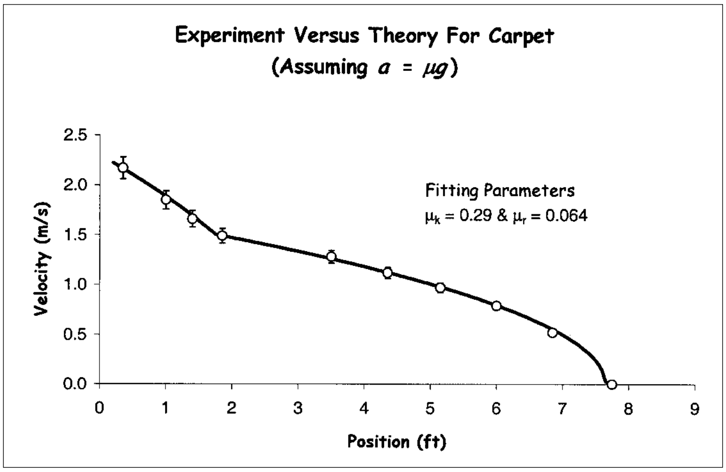

Fig. 1. Velocity data for a putted ball on a closely cropped carpet.

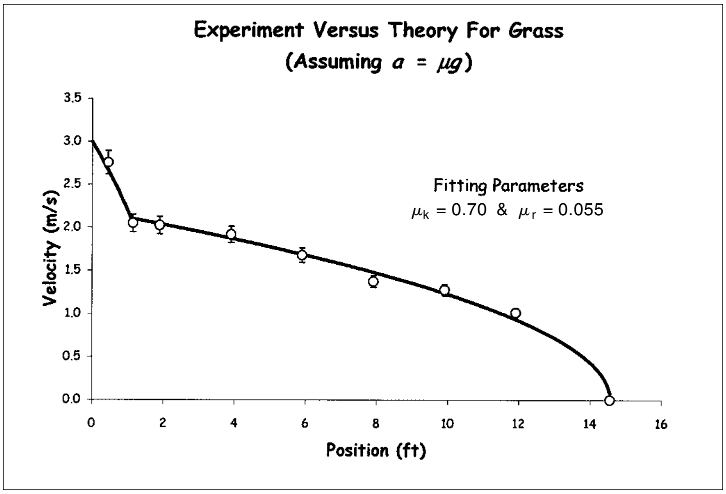

Fig. 2. Velocity data for a putted ball on a typical grass putting green.

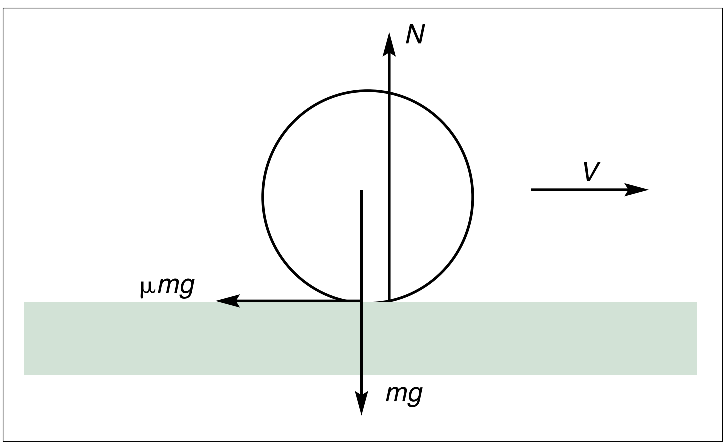

Fig. 3. The primary forces on a golf ball as it rolls over a putting green.

angular motion (see Fig. 3), the following is obtained:

$$m(V_o - V_f) = \mu_k \, mg\Delta t \tag{2}$$

and

$$\tfrac{2}{5}\, mR^2 \left(\frac{V_f}{R}\right) = \mu_k \, mgR\Delta t. \tag{3}$$

Solving these equations for Δt and then substituting into the following kinematics equation,

$$x_t = V_o\Delta t - \tfrac{1}{2}\,\mu_k\, g\Delta t^2, \tag{4}$$

leading to

$$x_t = \frac{12V_o^2}{49\mu_k\, g}\,. \tag{5}$$

In the previous derivation, it was assumed that the normal force acting on the golf ball produced no torque. This is not quite the case, and so the calculation actually underestimates x_t by about 10%. Actually, due to the asymmetry in the way the moving ball is supported by the blades of grass, the normal force acts along a line of action that is just slightly ahead of the ball's center of mass as shown in Fig. 3. This asymmetry is associated with encountering and deforming new blades of grass as the ball moves along its path. If the normal force did not act in this manner, you would have a rather paradoxical situation in which the force of rolling friction would reduce the ball's linear velocity while simultaneously producing a torque that would increase its angular velocity. The normal force produces a torque that is opposite to the frictional torque. This leads to a net torque that does indeed reduce the angular velocity of the ball as its linear velocity decreases. The numerical simulation that is discussed at the end of this paper properly accounts for this effect. As the ball comes to rest, it settles into the grass, and the supporting forces of the individual blades produce a symmetric deformation patch under the ball. In this case, the normal force acts along a line of action that passes up through the ball's center of mass. When the ball is at rest, the normal force no longer produces a torque.

From an energy point of view, there is mechanical energy lost due to both the kinetic friction of adjacent blades rubbing against each other as the ball rolls over them and to mechanical hysteresis associated with the deformation of individual blades. Before pure rolling begins, there would be additional energy loss associated with the kinetic friction between the ball and the grass as the ball skids over the surface.

Experimentally, the easiest way to determine the coefficient of rolling friction for a given putting green is to strike the golf ball with the putter so that the ball will roll 20 ft or so over a flat and level portion of the green. Direct the ball toward a marker that has been placed on the green out about 5 ft. This will assure that the ball has commenced pure rolling by the time it gets to this marker. Measure the distance D from the marker to where the ball comes to rest and the elapsed time T from the marker to the moment of rest. Then:

$$\mu_r = \frac{2D}{gT^2}. \quad (6)$$

Note: The Professional Golfers' Association (PGA) measures the frictional characteristics of a putting green with a device called a Stimpmeter.[4] This is little more than a standardized incline that is used to start the ball rolling at a given speed without skidding.

It would be possible to measure the kinetic coefficient of friction too by simply dragging a sled supported by two golf balls that have been glued to the sled so that they cannot roll. A spring scale can be used to measure the force F_c required to move the sled over a level section of the green at a constant velocity. Then

$$\mu_k = \frac{F_c}{mg}. \quad (7)$$

In this paper, both the kinetic and rolling coefficients of friction are treated as fitting parameters in Eqs. (8) and (9). The best-fit values are indicated in Figs. 1 and 2.

$$V = \sqrt{(V_o^2 - 2\mu_k g x)} \quad \text{for } x \le x_t \quad (8)$$

$$V = \sqrt{(V_o^2 - 2\mu_k g x_t - 2\mu_r g(x - x_t))} \text{ for } x \ge x_t \quad (9)$$

The velocity-versus-position data, in Figs. 1 and 2, show rather clear transitions from skidding to pure rolling.

Getting the ball to the hole is only part of the problem for a golfer. See Holmes' article[3] in which he determines the likelihood of the ball being "captured" by the hole once it gets there. If the ball reaches the hole at a speed of greater than about 1.6 m/s, it has virtually no chance of being captured. The Microsoft Excel putting simulation mentioned at the end of this note incorporates this aspect of putting.

For an interesting article on another type of friction associated with rolling, see Stepp's article.[5] He shows that a wheel with an axle has an effective coefficient of rolling friction, which is just the kinetic coefficient of friction between the axle and its hub reduced by the IMA (Ideal Mechanical Advantage) of the wheel-axle system. In such a system, the usual assumptions that are invoked are that there is no significant deformation of either the wheel or the surface upon which it rolls (i.e., an ox cart rolling over a perfectly smooth, flat, and hard dirt road). For this case, the effective coefficient of "rolling friction" (note this is not pure rolling as in the case of the golf ball rolling over a deformable surface) is given by:

$$\mu = \frac{r}{R}\mu_k, \quad (10)$$

where r is the radius of the axle shaft and R is the outer radius of the wheel.

It is interesting to note that the time that the putter head is in contact with the ball during impact is, to a first approximation, given by the half-period of a mass-spring system:

$$T_c = \pi\sqrt{\frac{m}{k}}, \quad (11)$$

where m is the mass of the golf ball (~46 g) and k is the average "spring constant" of the golf ball (~500 N/mm for deformations up to about 5 mm).[6] Using these numbers gives a contact time of just under a millisecond. A few years back a well-known manufacturer of golf equipment stated in its television ads that the golf ball is in contact with the club face for less than a tenth of a second, including putts, during a typical round of golf. This would seem to be true.

By the way, the purple carpet at the local miniature golf course registered the equivalent of a 14 on the PGA's Stimpmeter! That corresponds

Table I. Sample input data for the Excel putting simulation.

α (deg)	β (deg)	V_o (m/s)	θ (deg)	Note
0	-2	4.1	6.5	Ball escapes
0	-2	3.7	10	Ball captured
+3	-2	3.2	10	Ball escapes
+3	-2	2.5	20	Ball captured
-3	-2	4.7	6	Uphill putt

to a coefficient of rolling friction of 0.04. Golfers would call that a "very fast" green.

An Excel putting simulation, written by the author, incorporates the results of this paper. The simulation allows the user to change the slope of the putting surface. Uphill and downhill putts with or without sidehill slopes can be simulated. The interested reader may download this putting simulation at http://ic.arc.losrios.edu/~perrys/.

In the Excel putting simulation, α is the downslope of the green (a negative value indicates an uphill putt), β is a measure of the sideslope of the putting surface, and θ is the aiming angle (θ = 0 corresponds to launching the ball directly at the hole). Some good examples of input data for the simulation are shown in Table I.

The simulation demonstrates what most golfers know from experience. For a putt of a given length on a given slope, uphill putts require relatively high launch speeds and are aimed more directly at the hole. Downhill putts require a gentler touch and are aimed less directly at the hole.

References

1. Golfers often use what is called a slight "forward press" of their hands presumably to help prevent both lofting the ball off the surface and to help avoid imparting any backspin.
2. For more on skidding, see A. Cochrane and J. Stobbs, *The Search for the Perfect Swing* (Lippincott, New York, 1968), pp. 128–131.
3. Brian W. Holmes, "Putting: How a golf ball and hole interact," *Am. J. Phys.* **59**(2), 129–136 (Feb. 1991).
4. Brian W. Holmes, "Dialogue concerning the Stimpmeter," *Phys. Teach*. **24**, 401–404 (Oct. 1986).
5. Richard Stepp, "Why wheels work," *Phys. Teach.* **20**, 550–551 (Nov. 1982).
6. We measured *k* for a golf ball by using a Tinius Olsen Universal Testing machine to compress the ball. To the extent that this approximation of simple harmonic compression of the golf ball is true, it can be concluded that the contact time between the ball and the club is the same for a putt ball that rolls only five yards or a driven ball that flies 300 yards.