The run of a golf ball

A. Raymond Penner

Abstract: The run, which includes both the bounce and the roll, of a golf ball landing on turf is modeled. The effect of launch speed, impact angle, backspin, and green firmness on the run for a variety of golf shots is considered. It is found that the dominant factor that determines the length of the run, in the case of drives, is the impact angle. It is also found that for high-lofted iron shots, where the golf ball is given sufficient backspin, the ball may, for firm enough greens, initially bounce forward before running backwards.

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Résumé : Nous modélisons la trajectoire d'une balle de golf atterrissant sur du gazon, en incluant le rebond et le roulé subséquent. Nous examinons les effets sur la longueur du coup de golf de la vitesse initiale, de l'angle d'impact, de la rotation inverse et de la dureté du sol. Pour un driver, nous trouvons que le facteur dominant est l'angle d'impact. Pour un fer élevé, avec un angle très accentué, la balle de golf acquiert beaucoup de spin inverse et peut rebondir d'abord vers l'avant avant d'effectuer un mouvement de recul.

[Traduit par la Rédaction]

1. Introduction

One of the more impressive shots in the game of golf is when a golf ball lands on the green and the ball, after initially bouncing forward, rolls back towards the pin. Unfortunately, for most of us, this shot seems limited to the abilities of a skilled golfer. This paper will consider the physics behind this shot as well as others. A model of the run of a golf ball, which consists of both the bounce phase and the subsequent rolling after landing, will be presented and the effects of launch speed, impact angle, backspin, and green firmness will be considered.

The general behaviour of a bouncing ball, with the golf ball used as an example, is considered in a book by Daish [1]. However, the balls, in that case, are modeled as bouncing off rigid surfaces, which leads to discrepancies with the real behaviour of balls bouncing off compliant turf. Specifically, in the case of a golf ball, observations indicate that the initial bounce of the ball is much higher than is predicted by the rigid surface model and that the horizontal velocity after the bounce phase is also significantly less. Daish was aware of these discrepancies and made mention of them in his discussion on the behaviour of a bouncing cricket ball. This author [2] has previously used Daish's rigid surface model to determine the run of a golf ball in the case of a drive, where the discrepancies were ignored as only the overall run distance was wanted and the modeled run distance was fitted, by using a large enough value for the coefficient of rolling friction, to experimental results. However, in general, to correctly model the bounce and the overall run of a golf ball, the effect of the compliant nature of turf needs to be taken into account.

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A.R. Penner. Physics Department, Malaspina University-College, 900 Fifth Street, Nanaimo, BC V9T 1H7, Canada (e-mail: pennerr@mala.bc.ca).

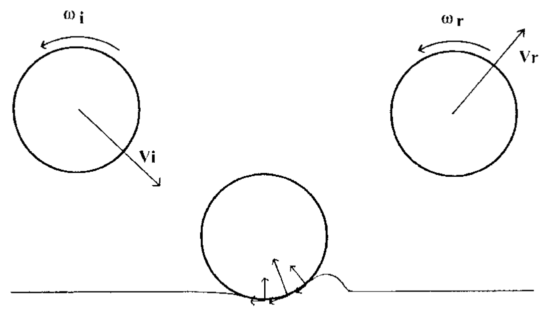


Fig. 1. The profile of the impact between a golf ball and compliant turf.

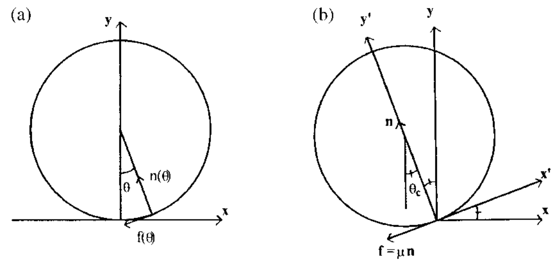


Fig. 2. (*a*) The components of the force acting at each contact point. (*b*) The components of the net force acting on the golf ball.

2. Model of the run

The behaviour of a golf ball bouncing on turf will, in general, depend on its landing velocity, the orientation and magnitude of its spin, and on the firmness of the turf. On impact the golf ball will start to both penetrate and to slip across the turf. The deformation of the turf can be large as is shown by the torn turf and the impact craters, with depths on the order of 1 cm, which are often seen on greens. Figure 1 shows the profile of the impact, with the golf ball impacting on the turf with a velocity of $\boldsymbol{v}_{\mathrm{i}}$ and with a backspin of ω_{i} and then rebounding with a velocity of $\boldsymbol{v}_{\mathrm{r}}$ and with a backspin of ω_{r}. The forces acting on the golf ball during contact will be distributed over the contact area. As an approximation, the contact forces that act on the golf ball will be taken to act only in the single plane that is shown in Fig. 1. Furthermore, the force acting at each contact point will be resolved into a component normal to the surface of the ball, $n(\theta)$, and a component parallel to the surface of the ball, $f(\theta)$, where θ locates the position on the ball's surface. This is shown in Fig. 2*a*. Treating $f(\theta)$ as being due to kinetic friction and equating it to $\mu n(\theta)$, where μ is the coefficient of kinetic friction between the golf ball and the turf, then results in the following components of the net force that acts on the golf ball during the collision:

$$\boldsymbol{n} = -\int n(\theta)\sin\theta\,\mathrm{d}\theta\boldsymbol{i} + \int n(\theta)\cos\theta\,\mathrm{d}\theta\boldsymbol{j} \tag{1a}$$

and

$$\boldsymbol{f} = -\left[\int f(\theta)\cos\theta\,\mathrm{d}\theta\boldsymbol{i} + \int f(\theta)\sin\theta\,\mathrm{d}\theta\boldsymbol{j}\right] \tag{1b}$$

$$= -\mu\left[\int n(\theta)\cos\theta\,\mathrm{d}\theta\boldsymbol{i} + \int n(\theta)\sin\theta\,\mathrm{d}\theta\boldsymbol{j}\right] \tag{1c}$$

As $\boldsymbol{n}\cdot\boldsymbol{f} = 0$ and $f = \mu n$, the distributed forces will, therefore, be equivalent to a single force with a component, $\boldsymbol{n}$, perpendicular to the ball's surface at a specific point, θ_{c} , and a component, $\boldsymbol{f}$, of magnitude μn parallel to the ball's surface at this same point. The problem then becomes equivalent to treating the golf ball as bouncing off of a rigid surface that is inclined at an angle of θ_{c} with respect to the horizontal. This model is shown in Fig. 2*b* where the x' axis is taken to lie along the equivalent rigid surface. The advantage of modeling the collision this way is that it allows the analysis by Daish, which was for a rigid surface, to be used. The position of the equivalent contact point, θ_c, and, therefore, the slope of the equivalent rigid surface will, in general, depend on the nature of the deformation of the turf. This would be expected to depend primarily on the impact speed and the impact angle of the golf ball as well as on the firmness of the turf.

The rigid surface model of a bouncing ball, as presented by Daish, considers two possible situations. The first case is where the ball slides over the surface throughout the impact. The frictional force is taken to act over the full duration of the collision and, as is shown by Daish, the rebound velocity components, $v_{rx'}$, $v_{ry'}$, and ω_r, of the ball will then be given by

$$v_{rx'} = v_{ix'} - \mu \left| v_{iy'} \right| (1 + e) \tag{2a}$$

$$v_{ry'} = e \left| v_{iy'} \right| \tag{2b}$$

and

$$\omega_r = \omega_i - \left(\frac{5\mu}{2r} \right) \left| v_{iy'} \right| (1 + e) \tag{2c}$$

where $v_{ix'}$ and $v_{iy'}$ are the impact velocity components with respect to the x'–y' frame of Fig. 2*b*, r is the radius of the ball, and e is the coefficient of restitution between the golf ball and the surface. The magnitude of the y' component of the impact velocity, $|\ v_{iy'}\ |$, is used in these and subsequent equations as $v_{iy'}$ is negative in the given reference frame. As neither $v_{rx'}$ or $v_{ry'}$ depend on ω_i in the above equations, the bounce length and height will be independent of the spin for this case.

The second case considered by Daish is where the frictional force is great enough to put the ball into a state of pure rolling during the collision with the surface. The frictional force will, therefore, fall to zero at some point during the impact and, as is shown by Daish, the final rebound velocities will be given by

$$v_{rx'} = \left(\frac{5}{7} \right) v_{ix'} - \left(\frac{2}{7} \right) r\omega_i \tag{3a}$$

$$v_{ry'} = e \mid v_{iy'} \mid \tag{3b}$$

and

$$\omega_r = -\frac{v_{rx'}}{r} \tag{3c}$$

The critical value of the coefficient of kinetic friction, μ_c, required to put the ball in a state of rolling during impact is shown by Daish to be given by

$$\mu_c = \frac{2(v_{ix'} + r\omega_i)}{7(1 + e) \mid v_{iy'} \mid} \tag{4}$$

For values of μ less than μ_c the golf ball will slide throughout the impact and (2) will apply while for values greater than μ_c the golf ball will roll out of the collision and (3) will apply. Daish suggests a value for μ of 0.40 for the specific case of a golf ball sliding on a green although no experimental results are given in support.

The coefficient of restitution, e, between a golf ball and turf and its dependence on impact speed has previously been measured by Penner [2]. It was found that the value of e decreased with increasing impact speed with the following function providing a good fit to the data:

$$e = 0.510 - 0.0375 \mid v_{iy'} \mid + 0.000903 \mid v_{iy'} \mid^2, \qquad \mid v_{iy'} \mid \leqslant 20 \text{ m/s} \tag{5a}$$

$$e = 0.120, \qquad \mid v_{iy'} \mid > 20 \text{ m/s} \tag{5b}$$

Although the data were collected on relatively soft turf this function will be used for the value of e in the model.

Using the above model, the only correction to Daish's rigid surface model that is required is the initial transformation from the fixed x–y reference frame to the x'–y' reference frame. This will be given by

$$v_{ix'} = v_{ix}\cos\theta_c - |\, v_{iy} \,| \sin\theta_c \tag{6a}$$

and

$$|\, v_{iy'} \,| = v_{ix}\sin\theta_c + |\, v_{iy} \,| \cos\theta_c \tag{6b}$$

Then after the rebound velocity components are determined, from either (2) or (3), the rebound velocity components, with respect to the x–y frame, are given by

$$v_{rx} = v_{rx'}\cos\theta_c - v_{ry'}\sin\theta_c \tag{7a}$$

and

$$v_{ry} = v_{rx'}\sin\theta_c + v_{ry'}\cos\theta_c \tag{7b}$$

As stated previously, the value of θ_c would be expected to depend primarily on the impact velocity components of the golf ball and on the firmness of the turf. This is an approximation as the spin of the golf ball would also be expected to play a role. Unfortunately, only one experimental measurement of the bounce of a golf ball on turf could be found in the scientific literature. Haake [3] recorded the impact between a golf ball and a typical golf green for a ball with an impact speed of 18.6 m/s, at an impact angle of 44.4° (the impact angles and rebound angles presented in this paper are all measured with respect to the vertical), and with backspin of 484 rad/s. The rebound speed was measured to be 5.1 m/s, at a rebound angle of 33.0°, and with a topspin of 145 rad/s. This result indicates that the frictional force was great enough to put the golf ball in a state of rolling during impact with the turf. Figure 3 shows the experimental result of Haake along with the result as determined using (3) in the case where $\theta_c = 15.4°$. As is seen, reasonable agreement is obtained between the bounce model and Haake's result with the model giving values for the rebound speed and rebound angle of 4.34 m/s and 34.1°, respectively, along with a topspin of 154 rad/s. The value of 15.4° for θ_c, which was used in the model, was determined to be the value that led to the best overall fit between the modeled rebound velocity components and the given experimental rebound velocity components. Using the values of $v_{ix'}$, $v_{iy'}$, and ω_i for this value of θ_c gives a value of 0.29 for μ_c along with a value of 0.14 for e. Therefore, since the golf ball was put into a state of rolling during impact, the coefficient of kinetic friction for the given green, from (3), must be greater than 0.29.

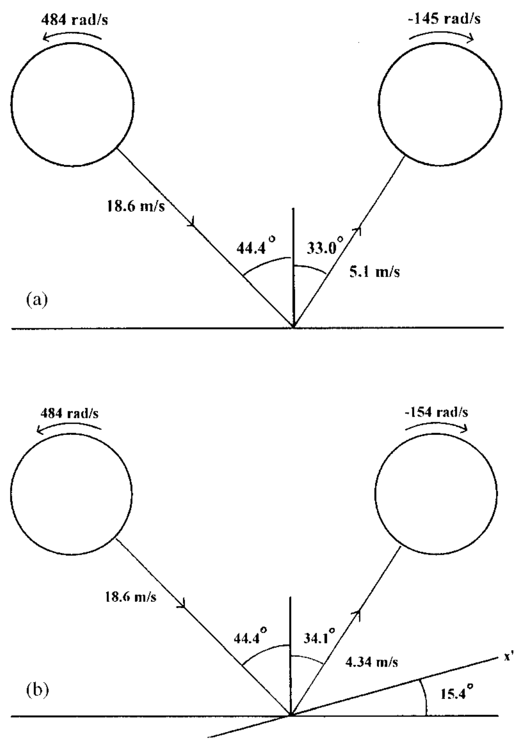
In general, it would be expected that the value of θ_c would increase with both the impact speed and the impact angle of the golf ball. As only one data point is given by Haake, it will be assumed that, as a first-order approximation, the value of θ_c increases linearly with both the impact speed and the impact angle. Therefore, using the value of 15.4° for θ_c for Haake's impact value results in the following general expression for the value of θ_c, which will be used in the run model:

$$\theta_c = 15.4°(v_i/18.6\text{m/s})(\phi/44.4°) \tag{8}$$

where $\phi = \tan^{-1} |\, v_{ix}/v_{iy} \,|$.

Measurements carried out by Hubbard and Alaways [4] on the interaction between a golf ball and a green showed that the final equilibrium resting position of a golf ball is approximately 2 mm below the top of the green's surface. Therefore, it is expected that the bounce model will break down as bounce

Fig. 3. (*a*) The experimental results of Haake [3]. (*b*) The results as determined by the given model with $\theta_c = 15.4°$.



heights approach the 2 mm value. For the results presented in this paper a value of 5 mm was set for the critical bounce height at which the transition occurs between the ball bouncing and the ball rolling along the green. The transition between bouncing and rolling will, of course, take place over several bounces and the 5 mm mark, which was used in the analysis, must be treated as a simplification of the actual behaviour of a real golf ball. Transition points between 1 mm and 10 mm were tested and the actual value had only a minor effect on the overall calculated run.

After the golf ball starts rolling it will decelerate because of rolling friction. Analysis of the behaviour of putts by Penner [5] shows that the acceleration of a rolling golf ball is given by

$$a = -\left(\frac{5}{7}\right)\rho_g g \tag{9}$$

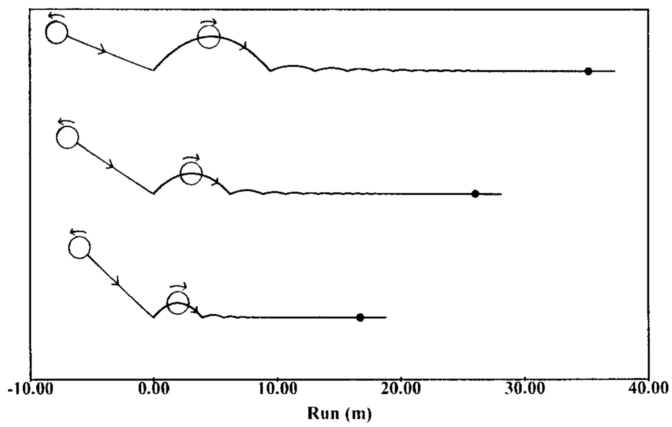
where ρ_g is the value of rolling friction. The value of rolling friction represents the relative displacement between the bottom of the golf ball and the equivalent contact point with the turf. For a typical golf green the value of ρ_g was found to be 0.131 [2]. Although this value was determined specifically for a green, and the value for a fairway would be expected, in general, to be greater, the value of 0.131 for ρ_g will be used throughout the analysis.

3. Results

3.1. The run for a drive

The major factor in determining the length of a drive is the speed of the club head during impact. The run of a golf ball in the case of a drive for various club-head speeds was therefore considered. The speed of a club head at impact for a typical drive ranges from approximately 35 m/s to 55 m/s for most golfers. Using the collision model between a club head and a golf ball, as given by Penner [2], for a club-head mass of 200 g; a dynamic loft (the angle the club face makes with the vertical at impact) of 13.3°; and

Fig. 4. The run of a golf ball for a drive with initial club-head speeds of 35 m/s (top), 45 m/s (middle), and 55 m/s (bottom). The • indicates the point where the golf ball stops bouncing and begins to roll.

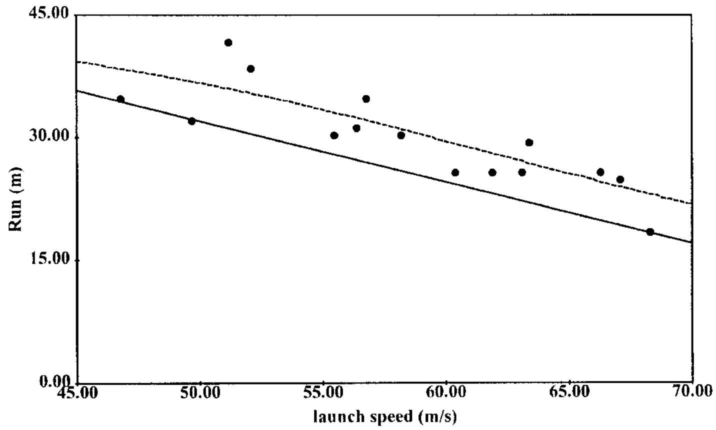


for this range of club-head speeds, it was determined that the launched golf ball leaves the club head at speeds ranging from 48.8 m/s to 74.3 m/s, at a relatively constant launch angle (measured with respect to the horizontal) of approximately 10.7° and with backspin ranging from 253 to 397 rad/s. Using these launch values the trajectories of the golf balls were determined. The trajectories were computed using the model as described by Penner [2] and the lift and drag coefficients presented by Bearman and Harvey [6]. The trajectories determined were then used to determine the landing-velocity components of the golf balls. It was found that over the given range of the club-head speeds, the landing speed of the golf ball was approximately constant with values ranging from 26.8 m/s for the 35 m/s club-head speed to 26.2 m/s for the 55 m/s club-head speed. The impact angle, however, varied greatly with the golf ball impacting at 68.0° for the 35 m/s club-head speed and at 46.2° in the case of the 55 m/s club-head speed. Also the amount of backspin at impact for the given club-head speeds varied from 224 rad/s for the 35 m/s club head up to 315 rad/s for the 55 m/s club head.

The landing velocities determined were then used to calculate the run of the golf ball using the given run model. First, using (8) and (6) the critical value μ_c, from (4), was found to range from 0.23 for the 35 m/s club-head speed to 0.18 for the 55 m/s club-head speed. Therefore, for an estimated value for μ of 0.40, as given by Daish, it can be concluded that the frictional force is great enough to put the golf ball into a state of rolling during the first bounce for all typical drives. Figure 4 shows the resulting runs corresponding to initial club-head speeds of 35 m/s, 45 m/s, and 55 m/s. The initial bounce height varies from 2.82 m for the 35 m/s club-head speed to 1.17 m for the 55 m/s club-head speed. Also, as is shown, the total run distance decreases with increasing club-head speed. It is somewhat surprising that for the harder hit drives the golf ball ends up with a smaller initial bounce, in both height and distance, and with a smaller overall run. The reason is due to the fact that although the launch speeds may be different, the actual landing speeds are approximately constant for the given range of club-head speeds. Therefore, the dominating factor in determining the height and length of the first bounce and the overall length of the run is the impact angle. Since the golf ball comes in at steeper angles for the higher initial launch velocities this leads to both a lower horizontal rebound velocity component and to a lower value for the coefficient of restitution, e. The result is a smaller first bounce and an overall smaller run.

Cochran and Stobbs [7] give empirical expressions for the carry and drive of a golf ball as a function of the launch speed of the golf ball. Therefore, an empirical expression for the run of a golf ball as a function of launch speed can be determined. Williams [8] also presented data that provide experimental values for the run of a golf ball as a function of the initial launch speed. Their results along with run distances, for various launch speeds, as determined by the given run model are shown on Fig. 5. As is seen, fairly good agreement is obtained between the model and the given empirical results. The slightly

Fig. 5. The empirical results of Cochrane and Stobbs [7] (—–) and Williams [8] (•) showing the dependence of the run of a golf ball on its launch speed compared with the results of the model (- - - -).



higher run values that are found with the model are most likely due to the landing and bouncing surface being taken to be flat. This will typically not be the case for a real fairway or green. However, as relatively good agreement with empirical results and observations is obtained, the model would appear to be a fair representation of the actual run of golf balls bouncing on turf.

3.2. The run for irons

The primary difference between the various irons is the loft of the club face. Club-face lofts vary typically from approximately 17° for a 1-iron to 46° for a 9-iron. The greater the loft of the iron the greater the launch angle of the golf ball and the greater the imparted backspin. For example, using Penner's [2] collision model, for a club-head speed of 40 m/s, and a typical iron club-head mass of 250 g, the golf ball is found to leave the club face at speeds ranging from 56.6 m/s for 1-irons to 42.5 m/s for 9-irons, and with respective launch angles ranging from 13.8° to 35.4°, along with backspins ranging from 372 to 915 rad/s. The corresponding landing speeds of the golf balls were again found to remain fairly constant over the range of irons, varying from 24.7 m/s for 1-irons to 23.8 m/s for a 9-iron. The impact angles, however, varied greatly with values ranging from 51.9° for the 1-iron to 34.0° for the 9-iron while the resulting backspin at impact ranged from 313 rad/s for the 1-iron to 809 rad/s for the 9-iron.

Consider first the runs that would be expected for the low to middle irons. Figure 6 shows the modeled runs for the 2-, 4-, and 6-irons. In all cases, the value of μ_{c} is less than 0.40 and the golf ball, as was found for drives, will, therefore, be in a state of rolling after it rebounds from the turf. Again the impact angle is the dominant factor in determining the size of the initial bounce and the overall length of the run. For these particular examples the initial bounce heights ranged from 0.97 m for the 2-iron, which had an impact angle of 42.8°, to 0.54 m for the 6-iron, which had an impact angle of 36.3°.

Now consider the runs from the higher lofted clubs. First, from (3a), it is seen that if

$$\omega_{\mathrm{i}} > \frac{5v_{\mathrm{i}x'}}{2r} \tag{10}$$

the x' component of the rebound velocity will be negative and the golf ball will bounce backwards. For the impact speed of 23.8 m/s and impact angle of 34.0° found above for the given 9-iron shot the critical value of ω_{i} is 903 rad/s. The actual value of 809 rad/s for the spin on landing that was calculated from the 9-iron trajectory, for the given 40 m/s club-head speed, is, therefore, near this value. In fact, the critical value of ω_{i} that is given by (10) is the value that will result in the golf ball bouncing backwards in the x'–y' frame. For values approaching this critical value, the golf ball may in fact have a positive

Fig. 6. Examples of the run of a golf ball for a 2-iron (top), 4-iron (middle), and 6-iron (bottom) shot. The • indicates the point where the golf ball stops bouncing and begins to roll.

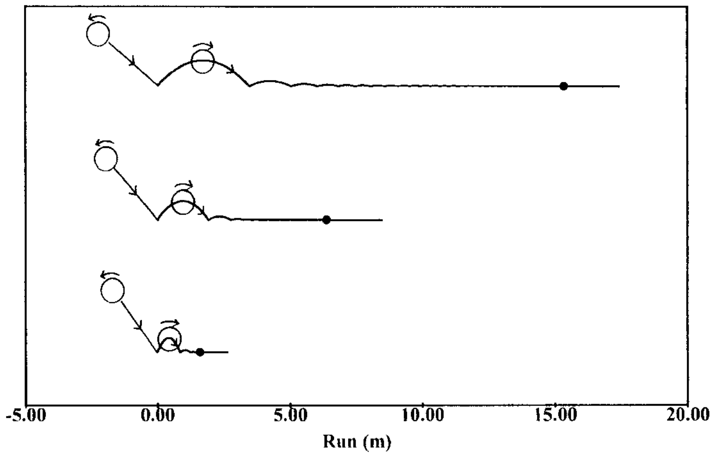
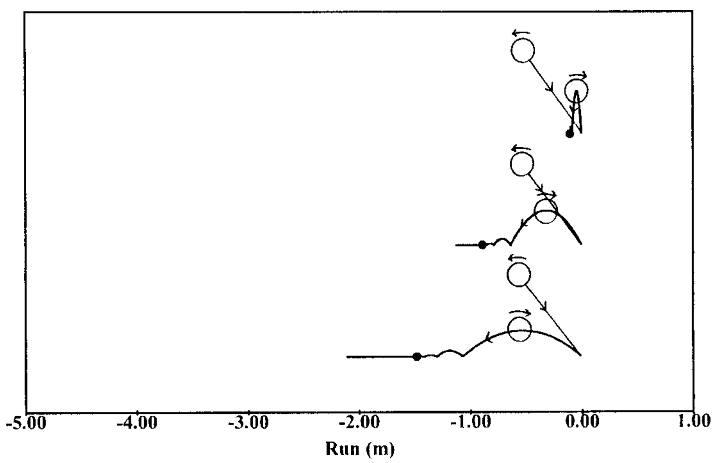


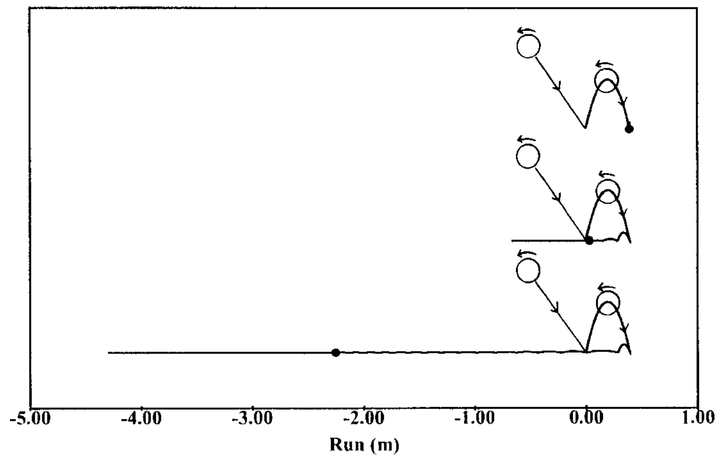
Fig. 7. Examples of the run of a golf ball for a 9-iron shot with landing spins of 809 rad/s (top), 1000 rad/s (middle), and 1200 rad/s (bottom). The • indicates the point where the golf ball stops bouncing and begins to roll.



$v_{\mathrm{r}x'}$ but a negative $v_{\mathrm{r}x}$. This is exactly what happens in this case and the golf ball, therefore, does end up bouncing backwards.

Professional golfers will often strike the golf ball with significantly greater club-head speed than the 40 m/s used in the above example. The golf ball will, therefore, in these cases, have much greater backspin on landing. In addition skilled golfers will often adjust their swing and their position relative to the ball to be able to increase the amount of backspin given the golf ball. The amount of backspin is the dominant factor in determining the behaviour of the run of a golf ball for the higher lofted irons. For example, Fig. 7 shows the effect backspin has on the run of a golf ball in the case of the above 9-iron shot with the impact speed of 23.8 m/s, impact angle of 34.0°, and with impact spins of 809 rad/s, which was calculated for the 40 m/s club-head speed, along with the higher backspin values of 1000 and 1200 rad/s. As is seen, the effect of the increased backspin is to increase the amount the golf ball bounces and rolls backwards. The values of μ_{c} for the 809, 1000, and 1200 rad/s backspins are 0.28, 0.33, and 0.38, respectively. Therefore, as a value of 0.40 for μ was used, the golf balls will be rolling out of the first bounce for all three cases and (3) will apply. However, as the balls initially bounce backwards they will have backspin when they strike the turf the second time. Equation (3) will again

Fig. 8. Examples of the run of a golf ball for a 9-iron shot to a firm green where $\mu > \mu_c$ with landing spins of 809 rad/s (top), 1000 rad/s (middle), and 1200 rad/s (bottom). The • indicates the point where the golf ball stops bouncing and begins to roll.



apply for this second bounce and the golf balls will come out of this bounce with top spin.

Consider now the effect of the firmness of the green on the run for the higher lofted irons. Specifically, the case where the value of μ is less than the critical value, μ_c. Figure 8 shows the runs for a golf ball that again is taken to impact at 23.8 m/s, at an impact angle of 34.0°, and with a backspin of 809 rad/s, 1000 rad/s, and 1200 rad/s. However, the coefficient of kinetic friction, μ, is taken to be equal to 0.25. This time the friction is not great enough to fully check the backspin and (2), therefore, applies for the initial bounce. As is seen, the golf ball initially bounces forward with backspin in all three cases before running backwards. Also, the height and length of the bounces are the same until the backspin is checked. This is due to, as previously mentioned, the fact that $v_{rx'}$ and $v_{ry'}$ in (1) do not depend on ω_i. In the case of the 809 rad/s backspin the amount the golf ball travels backwards after the first bounce is negligible while in the cases of the greater backspins of 1000 rad/s and 1200 rad/s the amount the golf ball runs backwards goes from 1.11 m to 4.74 m. Indeed, since this type of run is often seen when professionals play this would suggest values of μ closer to 0.25 apply to the greens that the tournaments are held on. However, it should also be pointed out that the equation for θ_c, (8), which was used to calculate the runs shown in Fig. 8 would, in general, need to be adjusted for greens with these lower values of μ. However, the general behaviour of a golf ball landing with high backspin on a firm green, as is shown in Fig. 8, would be expected to hold.

4. Conclusion

Treating the impact of a golf ball with compliant turf as being equivalent to an impact with a sloped rigid surface allowed for the analysis, as given by Daish, to be used. Results for the run are obtained from the model, which agree well with the actual behaviour of a golf ball during a game. For example, the modeled dependence of the run of a golf ball, in the case of drives, on the launch speed of the golf ball was found to agree, in general, with empirical results. It was also found that increasing the amount of backspin, for the higher lofted iron shots, increased the amount the ball runs backwards. In addition, it was found that for firmer greens, or smaller values of μ, golf balls with large amounts of backspin will initially bounce forwards before running backwards.

To further improve the run model that has been presented, additional measurements of landing and rebound velocities of golf balls impacting on turf would be required. This would allow for a more accurate determination of the dependence of θ_c on landing speeds and impact angles as well as on the firmness of the turf.

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