

# Sliding and rolling: the physics of a rolling ball

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J Hierrezuelo *Secondary School I B Reyes Católicos (Vélez-Málaga), Spain* and C Carnero *University of Málaga, Spain*

**We present an approach that provides a simple and adequate procedure for introducing the concept of rolling friction. In addition, we discuss some aspects related to rolling motion that are the source of students' misconceptions. Several didactic suggestions are given.**

Rolling motion plays an important role in many familiar situations and in a number of technical applications, so this kind of motion is the subject of considerable attention in most introductory mechanics courses in science and engineering. However, we often find that students make errors when they try to interpret certain situations related to this motion.

It must be recognized that a correct analysis of rolling motion requires a good understanding of a number of topics in mechanics. Several aspects have been shown to be the source of students' misconceptions. The first, and possibly the most important, is the one related to the role of the frictional force in rolling objects, as previously described (Shaw 1979, Salazar *et al* 1990, McClelland 1991, Carnero *et al* 1993). A second aspect refers to the difficulty for students in appreciating that translation and rotation occur simultaneously in rolling motion. This fact has recently been studied by Menigaux (1994), who rightly indicates that, in the case of a real body, deformation must be included in the analysis of the problem. Moreover, in the study of rolling motion

there is an extra difficulty: most students think that it is not possible for a body to roll without slipping unless there is a frictional force involved. In fact, when we ask students, 'why do rolling bodies come to rest?', in most cases the answer is, 'because the frictional force acting on the body provides a negative acceleration decreasing the speed of the body'. In order to gain a good understanding of rolling motion, which is bound to be useful in further advanced courses, these aspects should be properly clarified.

The outline of this article is as follows. Firstly, we describe the motion of a rigid sphere on a rigid horizontal plane. In this section, we compare two situations: (1) rolling and slipping, and (2) rolling without slipping. From the results obtained in this study, the case of a real body is considered. Here, the concept of rolling resistance is established.

## **Rigid sphere moving on a rigid horizontal plane**

Let us consider, for instance, a rigid sphere, solid and uniform, of mass  $M$  and radius  $r$ , which is thrown horizontally along a rigid horizontal surface with an initial velocity  $v_0$ . To study the motion described by the sphere we must consider two phases which are clearly different. In the first phase a frictional force ( $f$ ) acts on the sphere in the direction opposite to the motion of the centre of mass (CM) of the body, as

shown in figure 1. In this stage the sphere rolls and slips at the same time. The dynamic equations that describe this situation are

$$\Sigma F = Ma_{cm} \quad -f = Ma_{cm} \quad (1)$$

$$\Sigma \tau = I_{cm} \alpha \quad fr = I_{cm} \alpha \quad (2)$$

where  $a_{cm}$  is the acceleration of the CM,  $I_{cm}$  is the moment of inertia about an axis through the CM ( $I_{cm} = \frac{2}{5} Mr^2$ ), and  $\tau$  is the torque with respect to the CM. It is important to note that, as the sphere is rolling and slipping, we must calculate torques only about an axis passing through the CM. This question is often the origin of errors, since some authors advise the choice of the so-called instantaneous axis (axis passing through the point of contact between the rolling body and the surface) to describe the rolling motion, though the use of this axis is possible only for rolling bodies without slipping. Although in certain situations the choice of the instantaneous axis can facilitate the calculation, we have observed that it can also lead students into errors. For example, we have often found that students take the instantaneous axis even in the case of a body that rolls and slips at the same time. In our opinion there are two reasons why the use of the instantaneous axis to describe the motion of rolling bodies should not be advised. (i) Students tend to apply the same approach regardless of its applicability. (ii) With the use of the instantaneous axis students may forget the fact that translation and rotation occur simultaneously.

Note that  $f$  is the sliding frictional force (dynamic friction force), since slipping between the sphere and the surface occurs, hence this frictional force is given

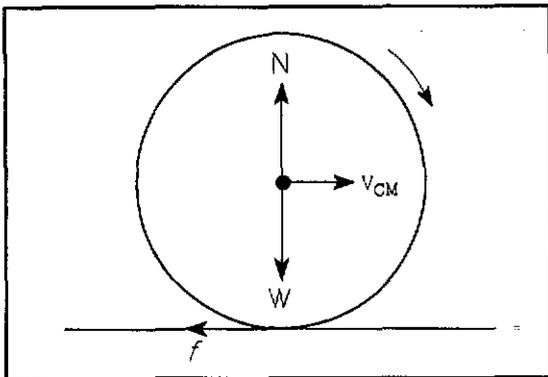


Figure 1. Rigid sphere rolling and slipping along a horizontal rigid surface.

by

$$f = \mu_k Mg \quad (3)$$

where  $\mu_k$  is the coefficient of kinetic friction between the two surfaces. From equations (1), (2) and (3) we obtain the acceleration of the CM, as well as the angular acceleration of the sphere:

$$a_{cm} = -\frac{\mu_k Mg}{M} = -\mu_k g \quad (4)$$

$$\alpha = \frac{5\mu_k Mg}{2Mr} = \frac{5\mu_k g}{2r} \quad (5)$$

On the other hand, as the acceleration of the CM and the angular acceleration are constants during this phase, the corresponding kinematic equations can be used to obtain the linear speed ( $v_{cm}$ ) and the angular speed ( $\omega$ )

$$v_{cm} = v_0 - \mu_k g t \quad (6)$$

$$\omega = \frac{5\mu_k g}{2r} t \quad (7)$$

where we have assumed that the sphere was thrown without initial angular speed. One should note, from the expressions previously derived, the important fact that in this first phase of the motion the linear and rotational quantities are not related through  $v_{cm} = r\omega$  and  $a_{cm} = r\alpha$ , since the sphere rolls and slips at the same time. This point is often the source

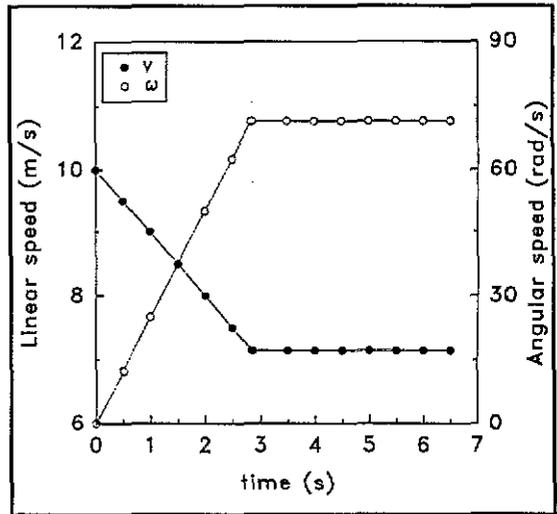


Figure 2. Linear and angular speeds as functions of time for a rigid sphere thrown along a horizontal rigid surface. (See the text for conditions of the motion.)

of students' errors, and therefore it must be made clear that, although there is a connection between translation and rotation,  $v_{cm} \neq r\omega$  and  $a_{cm} \neq r\alpha$  because of sliding.

From equations (6) and (7) it can be observed that, while the sphere is acquiring angular speed, its linear speed gradually decreases. At the point when  $v_{cm} = r\omega$ , the sphere thereafter rolls at constant speed without slipping. At this moment the frictional force disappears, and the sphere begins the second phase of its motion. In this second stage the sphere describes a uniform motion. In figure 2 we have plotted  $v_{cm}$  and  $\omega$  as functions of time for a sphere of radius 10 cm, which is thrown horizontally with an initial speed of  $10 \text{ m s}^{-1}$  along a horizontal plane, and assuming a coefficient of kinetic friction of 0.1. In this figure the two phases of the motion are clearly shown. The first phase is described by equations (6) and (7), and here a frictional force is involved. The second phase corresponds to uniform motion, and there is no frictional force. Our experience reveals that this is a delicate point in the analysis of rolling motion.

In general, students raise two questions: (i) how can the frictional force disappear?, (ii) if the frictional force disappears, what causes the torque providing the rotation? The first question derives from a poor understanding of the concept of frictional force; in this respect we must remember that a frictional force appears (a) when there is a relative motion between two surfaces or (b) when there is a force that attempts to cause relative motion, though without achieving it. Neither of the two conditions occurs in the second phase of the motion. Note that when a rolling body rolls without slipping, the point of contact of the body with the surface is always instantaneously at rest with respect to the sustaining surface. The idea that a torque is necessary to maintain rotation is a very common misconception of Newton's second law for rotation. Here, we must emphasize again that a torque acting on a body causes a *change* in its angular speed.

### Rolling friction

From the preceding section it can be concluded that, once the sphere reaches the pure rolling state (rolling without slipping), the motion continues indefinitely with a constant speed. However, our experience shows that, sooner or later, the body stops. To explain

this fact we must analyse the problem from a real point of view. Hitherto we have considered that both the body and the surface are perfectly rigid, which is not a real situation. In general, we must assume that the rolling and/or the sustaining plane deform slightly, in such a way that there is not a contact point between them, but a contact area. Although, in general, the deformation is produced in both surfaces, in most cases we can neglect the deformation of one of them. In this way we can divide the problem into two parts: a deformable body rolling along a rigid surface, and a rigid body rolling on a deformable surface.

**Deformable body rolling freely on a rigid horizontal surface.** This would be the case, for instance, of an automobile tyre when it is not under traction. An interesting study on this subject, including more complex situations, has recently been published (Tabor 1994). In this case an asymmetric deformation takes place, in such a way that the normal force exerted by the plane (i.e. horizontal surface) on the tyre now acts at a point slightly displaced from the theoretical contact point. Figure 3 shows such a situation. As a consequence, the normal force exerts a torque about the CM (see figure 3) opposed to the tyre rotation, which causes a reduction in the angular speed of the tyre. This torque is given by

$$\tau_f = \rho N \quad (8)$$

where  $\rho$  is the displacement of  $N$  with respect to the action line of  $W$ .  $\rho$  depends essentially on three factors: (i) the elastic properties of the rolling body, (ii) its radius and (iii) the effective load that the tyre bears. Because the torque opposes the tyre rotation, it is referred to as rolling friction and is

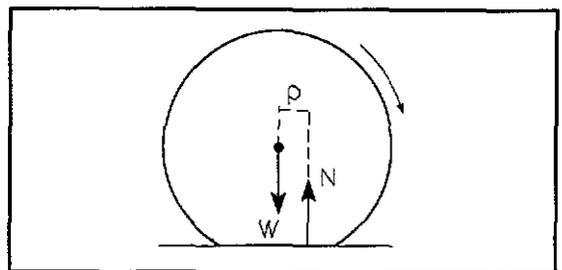


Figure 3. Diagram showing the forces acting on a deformable rolling object moving on a horizontal rigid surface.

usually many times smaller than the sliding friction; therefore balls and rollers are widely used in many technical applications to reduce the friction between surfaces.

**Rigid rolling body moving freely on a deformable horizontal surface.** This is the case of a billiard ball moving on a cloth-covered billiard table. Here the deformation of the ball can be neglected in relation to the deformation of the cloth. The situation that we must analyse is shown in figure 4(a). Now the reaction force  $R$  will be normal to the surface of contact, and will be applied at a point ( $A'$ ) slightly above the theoretical point ( $A$ ). As a consequence,  $R$  is not vertical and can be resolved into two components—a horizontal component ( $R_x$ ) and a vertical component ( $R_y$ ). These two components are shown in detail in figure 4(b).

The equations that govern the motion of the ball, according to figure 4(b), are

$$-R_x = Ma_{cm} \quad (9)$$

$$R_y - W = 0 \quad (10)$$

$$R_x h - R_y \delta = I_{cm} \alpha \quad (11)$$

where torques have been taken about the  $cm$ , and the moment of inertia is expressed about an axis through the same point. If the ball rolls without slipping ( $a_{cm} = r\alpha$ ) then, taking into account the  $I_{cm}$  value ( $\frac{2}{5}Mr^2$ ) and assuming small deformations ( $h \approx r$ ), it can be immediately deduced from equations (9) and

(11) that

$$R_y \delta = \frac{7}{5} R_x r \quad (12)$$

which indicates that

$$R_y \delta > R_x r. \quad (13)$$

In this manner, the deceleration of the ball is justified by the contribution of two effects: (1) the action of the horizontal component ( $R_x$ ) opposed to the velocity of the  $cm$  of the ball, and (2) a resultant torque that provides a deceleration of the rotation. In summary, the force  $R$  plays the role of a rolling resistive force, which explains the deceleration of the ball.

On the other hand, equation (12) can also be expressed as

$$\frac{R_x}{R_y} = \frac{5}{7} \frac{\delta}{r} \quad (14)$$

where  $\delta/r = \sin \theta$  (see figure 4(b)). Assuming small deformations ( $\sin \theta \approx \theta$  and  $\sin \beta = \beta$ ) and taking into account that  $R_y = W$ , we obtain

$$R_x = R\beta = \frac{5}{7} M g \theta \quad (15)$$

so that the rolling resistive force  $R$  is a constant force, which is determined by the deformation of the surface, but independent of the velocity of the ball.

Moreover, we think that it is possible to give a further argument to explain how a rolling body comes to rest. The velocity of any point ( $P$ ) of the rolling body is the

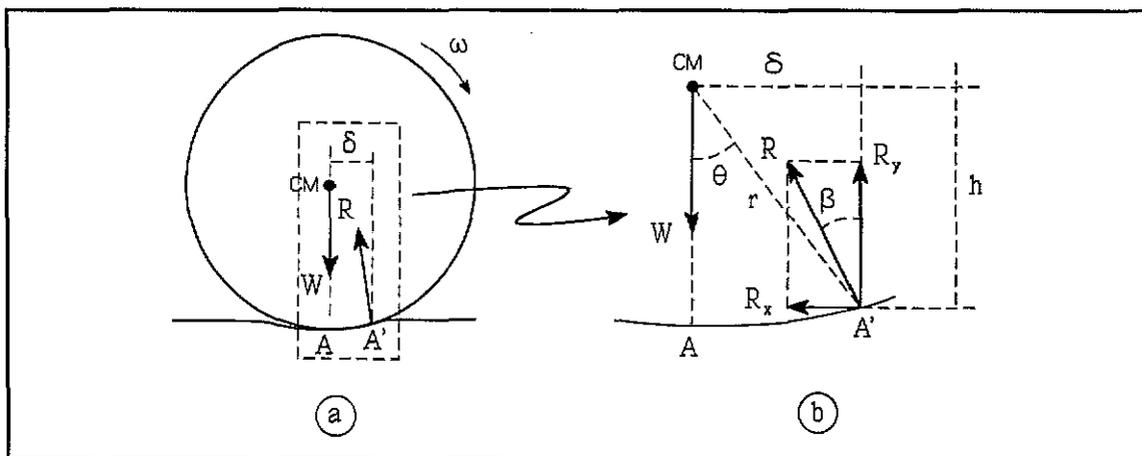


Figure 4. (a) Rigid sphere on a deformable surface. (b) Detail of the distribution of forces acting on the sphere.

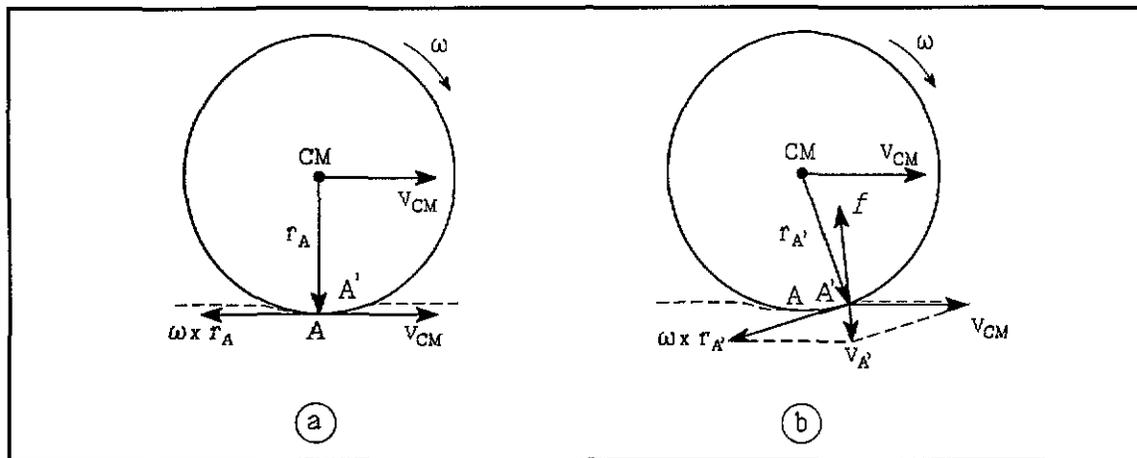


Figure 5. Rigid sphere on a deformable surface. While the velocity of the point A is zero (a), the point A' has a velocity ( $v_{A'} \neq 0$ ) with respect to the surface (b).

vector sum of two velocities: the velocity of the CM ( $v_{cm}$ ) relative to the surface plus the tangential velocity ( $\omega \times r_p$ ) of the point relative to the CM, that is

$$v_p = v_{cm} + \omega \times r_p. \quad (16)$$

For this reason, while the velocity of the point A is zero (see figure 5(a)), the velocity of the point A' is not zero, but is finite relative to the surface as illustrated in figure 5(b). Since there is a relative motion between the point A' and the surface, it seems correct to consider a sliding frictional force acting at the point A'. Given that the sliding frictional force is always opposed to the relative motion between the two surfaces, it would be valid to suppose that this frictional force ( $f$ ) has a direction opposite to the velocity of the point A', as shown in figure 5(b). In this manner, the presence of this sliding frictional force ( $f$ ) can also explain the decrease of both the angular speed and the translational speed of the body.

In our opinion, this last argument must be taken into account as well as the effect caused by the action of the force exerted by the surface on the body, as established above. An important feature in rolling motion consists in appreciating the fact that the absence of a sliding frictional force, in the case of a rigid body rolling along a rigid surface with a constant speed, is due not to the rigidity of the bodies but to the fact that the point of contact of the body with the surface is instantaneously at rest. Therefore, when a deformation is produced, the velocities of the points of contact between the body and the surface are not zero, and as a consequence a

sliding frictional force appears, which contributes to stopping the body.

### Concluding remarks

The treatment presented in this paper may contribute to a better understanding of rolling motion, and we think that it is highly advisable in a physics introductory course. In particular, the use of the approach given here can help to essentially clarify two points: (1) that a frictional force is not always involved in a rolling motion, and (2) that a complete analysis of rolling motion assumes a deformation in the rolling surface and/or on the sustaining surface. With regard to this last aspect, we think that the discussion of the concept of rolling friction could form an adequate introduction, which could help students in the analysis of more complex physical phenomena.

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## NEW APPROACHES

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