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ORIGINAL ARTICLE

Understanding the role of shaft stiffness in the golf swing

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Abstract Theoretically, shaft stiffness can alter shot distance by increasing clubhead speed or altering clubhead orientation at impact. A 3D forward dynamics model of a golfer and flexible club simulated the downswing. A genetic algorithm optimized the coordination of the model's muscles (four torque generators) to maximize clubhead speed. The maximum torque output and maximum rate of torque development from the torque generators were varied to simulate the swing of golfers that generate different clubhead speeds. Four shafts of varying stiffness (flexible, regular, stiff, and completely rigid) were entered into these simulations to examine the role that shaft flexibility had on clubhead speed and orientation at impact. Shaft stiffness was found to have a meaningful effect only on clubhead orientation (dynamic loft and dynamic close) at impact. There was no evidence to support the premise that matching the stiffness properties of the shaft with the golfer would improve clubhead speed.

Keywords Golf · Shaft flexibility · Computer simulation · Optimization · Three-dimensional · Forward dynamics · Genetic algorithm

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1 Introduction

Over the years, the golf club has gone through many modifications to improve performance. In response to the evolution of design changes, the governing bodies (United States Golf Association and The R&A) have introduced regulations on golf equipment aimed at protecting the best interests of the game [1]. Golf club manufacturers are now focusing on new strategies to attract consumers such as customizing the stiffness of a golf club's shaft to an individual's swing. The stiffness of a shaft can, in theory, exert its influence on the resulting ball flight in two ways. The first involves the shaft's ability to store and subsequently release strain energy which could result in an increase in clubhead speed. The second is by altering the orientation of the clubhead relative to the ball at impact. The orientation of the clubhead will affect the distance the ball travels by changing the launch angle relative to the horizontal, the direction of ball flight, and the spin rate of the ball.

Prior to impact with the ball, the shaft can be measured bending about three orthogonal axes fixed to the grip end of the club. Deflection along the *Y* axis represents lead/lag motion (Fig. 1a), while deflection along the *X* axis represents toe-up/toe-down motion (Fig. 1b). Twisting about the longitudinal, *Z,* axis of the shaft can also occur. Compared to the magnitude of deflection about the other axes, twisting about the longitudinal axis has a negligible influence on both the orientation of the clubhead and its velocity at impact, and therefore, will not be considered in this paper [2]. Butler and Winfield [2] measured peak deflection values in the lag direction as large as 7 cm, and peak deflections in the toe-up direction greater than 15 cm. In their study, three golfers swinging the same club at 46 m/s, produced toe-down deflections at impact that ranged from approximately 0.5–5 cm, while lead deflections at impact ranged from approximately 0.3–4 cm. Mather and Cooper [3] found that, for a 'good player' swinging a driver, both the lead and toe-down deflections at impact can be as large as 5 cm. Horwood [4] determined that, for one golfer swinging a stiff flex shaft with a clubhead speed of 42.5 m/s at impact, the clubhead moved through 12.7 cm from its maximum lagging position into its maximum leading position. For a group of golfers with an average clubhead speed of approximately 46 m/s, Nesbit [5] measured an average lead deflection of approximately 6 cm at impact. Perhaps the most cited study regarding the role of the shaft in the golf swing is that of Milne and Davis [6]. A graph of their computer simulation values showed 'in-swing-plane' deflection values exceeding 10 cm. Since Milne and Davis employed a 2D model, it is assumed that 'in-swing-plane' refers to a blend of lead/lag and toe-up/down deflection. Based on the findings in the literature, it appears that the shaft does bend considerably during the swing.

Researchers have attempted to quantify the effect of shaft bending on clubhead speed. Nesbit [5] stated that shaft flexibility plays an important part in generating clubhead velocity through correct timing of the recoil of the shaft. The speed generated from the recoil of the shaft near impact is referred to as kick velocity [2]. Mathematically, kick velocity is the derivative of lead/lag deflection with respect to time. Butler and Winfield [2] calculated kick velocities at impact that ranged from 2.27 to 2.48 m/s. Horwood [4] made similar findings and determined that the maximum kick velocity was 5% (2.01 m/s) of the total clubhead speed (42.5 m/s). Contrary to these findings, Milne and Davis [6] concluded that shaft flexibility does not play an important dynamic role in the golf swing. It is unclear from their published work how this conclusion was reached. As stated previously, their results demonstrated large clubhead deflections during the downswing ($\sim$10 cm), which implies the storage of strain energy and the possibility of kick velocity adding to the overall clubhead speed. However, there was no mention of kick velocity in the paper or how shaft bending during the swing affects clubhead speed. MacKenzie [7] provided a critique of the simulation methods used by Milne and Davis [6], which called into question their two-dimensional model's ability to evaluate shaft flexibility. Recently, Worobets and Stefanyshyn [8] experimentally examined the influence of shaft stiffness on clubhead speed by having 21 golfers execute ten swings with each of five shafts of varying stiffness. For the majority of the golfers (12/21), shaft stiffness was reported to have no effect on clubhead speed; however, shaft stiffness did have an effect for nine of the participants. The fact that all of their tested golfers demonstrated "remarkable swing consistency" contributed to the researchers' inability to explain the ambiguous results. Without information on golfer hand speed, it cannot be definitively determined whether changes in clubhead speed were a result of altered shaft dynamics or modified golfer kinematics. The exact methods of filtering and interpolating the kinematic data to determine clubhead speed at impact were not reported. These procedures are not trivial as the clubhead would experience high frequency movement (due to ball impact) at the precise time when the swings' representative clubhead speeds were measured.

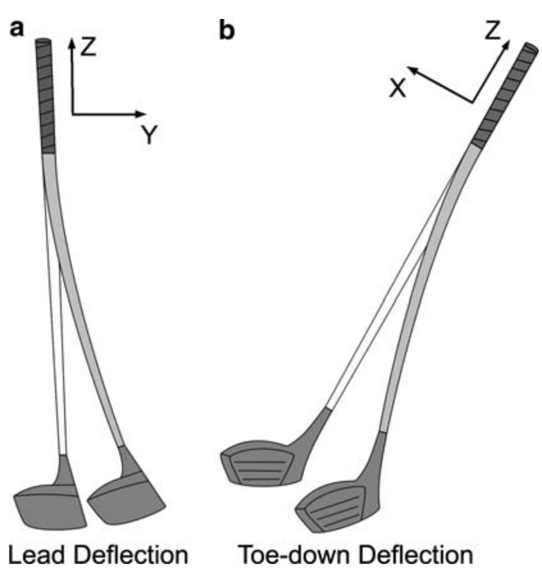

Fig. 1 The modeled shafts were capable of deflecting about two axes. **a** Deflection along the *Y* axis represents lead/lag motion. **b** Deflection along the *X* axis represents toe-up/toe-down motion

Although it is generally accepted that the orientation of the clubhead relative to the ball is altered by the shaft bending near impact, few studies have attempted to quantify the effects. Mather and Cooper [3] stated that depending on the geometry of the shaft, a lead deflection of 5 cm can result in a 5° increase in the loft of the club. They refer to this added loft as dynamic loft (Fig. 2a). Horwood [4] explained that increasing the lead deflection at impact would increase the dynamic loft at impact and result in a higher ball trajectory. Dynamic close also occurs as a result of clubhead deflection and is a close in the face of the clubhead relative to the intended clubhead direction (Fig. 2b). Although not explicitly reported by any of the previously mentioned researchers, bending in the toe-up/toe-down direction may also alter ball flight.

The purpose of this paper was to gain an understanding of the role that shaft stiffness plays during the golf swing. This was accomplished through the use of mathematical modeling and optimized computer simulation techniques.

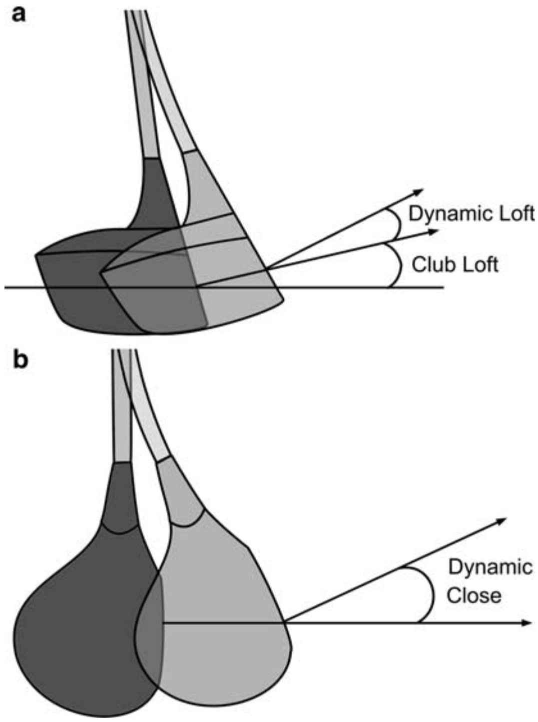

Fig. 2 **a** Dynamic loft is the change in nominal club loft that results from clubhead deflection. **b** Dynamic close also occurs as a result of clubhead deflection and is a close in the face of the clubhead relative to the intended clubhead direction

2 Methods

2.1 Model description

A representative mathematical model of a golfer was constructed using a six-segment (torso, arm, and four club segments), 3D, linked system (Fig. 3). The golfer portion of the model had four degrees of freedom. The model was capable of torso rotation, horizontal abduction at the shoulder, external rotation at the shoulder, and ulnar deviation at the wrist. Four muscular torque generators, which adhered to the force–velocity and activation rate properties of human muscle, were incorporated to add energy to the system. The four segments of the modeled club were connected in series by rotational spring-damper elements (Fig. 3). The hand and most proximal club segment were combined to represent a single segment, *Club_Proximal* [5]. The shafts were capable of deflecting about two axes (Fig. 1). Further details on model development and parameters have previously been presented [9].

Three versions of the same base model were used in this study. They differed only with regards to the constraint parameters governing the maximum torque output from the four torque generators. This allowed the role of shaft flexibility to be evaluated for golfers that generate three different levels of clubhead speed (i.e. Golfer-Slow

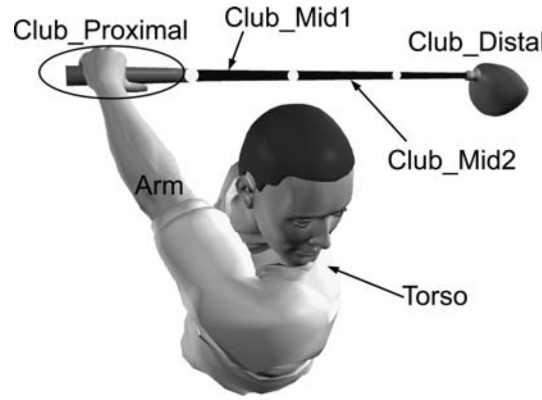

Fig. 3 The initial configuration for the 3D, six-segment model used to simulate the downswing. Note that the most proximal club segment was comprised of both the golfer's hand and grip of the club

~35 m/s, Golfer-Medium ~43 m/s, and Golfer-Fast ~50 m/s). These clubhead speed values represent the minimum, average, and maximum clubhead speeds measured by Brown et al. [10] on a group of 40 male golfers (age 20–59; handicap 14 ± 8).

2.2 Determining shaft stiffness and damping parameters

Separate stiffness constants for each of the three interconnecting springs were experimentally determined so that three shafts of varying stiffness could be employed in the model. To achieve this, three identical metal drivers were fitted with shafts of different stiffness (flexible, regular, and stiff) by a club professional with 30 years of experience. Each constructed club was measured to have a D1 swingweight. Once constructed, each club was rigidly secured in a vise so that the first 30 cm of the grip end was completely rigid (Fig. 4). This simulated the modeled club which was completely rigid for the first 30 cm. Markers were placed on the shaft so as to identify the segments defined by the mathematical club model. A 1 kg mass was suspended

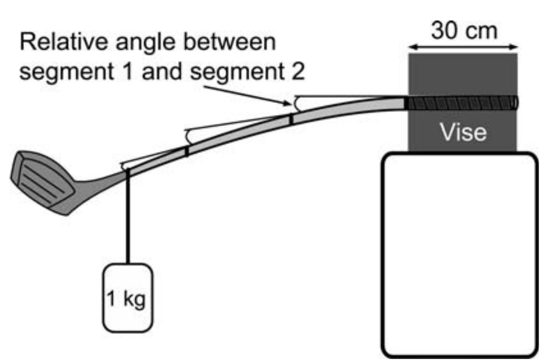

Fig. 4 Experimental set-up for determining shaft stiffness

from the club, at the point where the shaft inserts into the hosel, resulting in shaft deflection. The deflection was video recorded and the coordinates of the markers were determined using the motional analysis software package HU-M-AN™. From these coordinates, the relative angles between adjacent links were determined. This procedure resulted in three sets of three angles, with each set of angles representing a different shaft.

This experimental setup was duplicated using the mathematical model of the club. Using an optimization scheme, stiffness constants for each of the three spring elements were adjusted until the relative angles between segments in the mathematical model matched a set of relative angles determined experimentally. This procedure was repeated for each of the three clubs measured experimentally resulting in stiffness parameters for Flexible, Regular, and Stiff club shafts (Table 1). For reference purposes, a club (Rigid) with a fourth level of shaft stiffness was developed. The shaft was modeled as a single rigid link by incorporating motion constraints at the three joints that connected the four club segments.

Accurately measuring and consequently modeling the damping brought about by the connection of club with the soft tissue in the hands of the golfer is a difficult task [3, 6, 11]. Determining a damping coefficient experimentally, would require that the club be gripped by a golfer's hands and put into an oscillation. Unfortunately, a golfer cannot simulate the grip characteristics that are applied during the swing in the type of experimental set-up that would be required to determine a damping coefficient. Therefore, a level of damping was chosen that resulted in the best agreement between simulations and previously published live golfer testing results [2, 12, 13]. Nesbit employed a similar modeling technique to represent the damping present during the downswing [5]. The damping coefficient that provided the best level of agreement was 10 (Nm/rad/s). This meant for a given axis of rotation, a rotational spring-damper element took the form:

$$\text{Torque} = (-K\theta) - (C\omega) \quad (1)$$

where K is the stiffness coefficient, θ is the relative angular displacement of the distal segment, C is the damping coefficient, and ω is the relative angular velocity of the distal segment.

Table 1 Stiffness coefficients for the rotational springs of the simulated driver shafts (Nm/rad)

Club	Proximal	Middle	Distal
Flexible	501	135	81
Regular	536	159	87
Stiff	625	194	148

2.3 Model optimization

The goal of the computer simulation was to maximize horizontal clubhead speed at impact with the golf ball. The control variables were the onset and duration times for the four torque generators. This resulted in a total of eight control variables that were optimized to determine maximum horizontal clubhead speed at impact. The reader is referred to the first paper in this series for a more detailed description of the optimization methodology [9]. The optimization process was repeated for each golfer-club model incorporating each level of shaft stiffness.

3 Results

A clear pattern emerged in the maximum lag deflection of the clubhead across simulations (Table 2). As swing speed increased from Golfer-Slow to Golfer-Fast, the magnitude of lag deflection increased. Also, within each level of swing speed, the magnitude of lag deflection increased as shaft stiffness decreased. The reader is referred to Fig. 9 in the first paper in this series for a depiction of the time history of shaft deflection in all directions during the downswing [9]. It should be noted that the presence of the 'out of swing plane' shaft deflection would not be observed with a 2D model. The observed deflection also confirms the storage of strain energy in the shaft and suggests the possibility of the shaft facilitating clubhead speed if the transference into kinetic energy can be properly timed. The same pattern emerged between club shaft stiffness, swing speed, and lead deflection at impact, as with the lag data above (Table 2). It was observed that, for every optimized swing, the maximum lead deflection occurred at impact. The maximum toe-down deflection values also occurred nearly simultaneously (within 0.002 s) with impact during all optimized simulations.

The dynamic loft (Fig. 5) and dynamic close data (Table 2) revealed the same pattern across stiffness conditions as the lead deflection data. This finding was expected since dynamic loft and dynamic close are dependent upon the amount of lead and toe-down deflection present at impact [7]. The Golfer-Fast\Club-Flexible model produced the largest dynamic loft (6.27°) and dynamic close (5.17°), while Golfer-Slow\Club-Stiff generated the smallest dynamic loft (4.42°) and dynamic close (4.01°). It should be noted that these values of dynamic close were generated without any ability of the shaft to twist about its longitudinal axis.

Regardless of shaft stiffness, and within each level of relative golfer speed, there were no meaningful differences in clubhead speed when the shaft was able to flex (Fig. 6). The largest difference in clubhead speed (0.08 m/s) for a

Table 2 Golf club dynamics during the nine simulated swings

Golfer	Club	Max lag (cm)	Lead at impact (cm)	Max toe-up (cm)	Toe-down at impact (cm)	Dynamic close (°)	Kick velocity (m/s)
Slow	Stiff	2.70	5.74	7.19	1.79	4.01	4.89
	Regular	2.82	5.78	8.60	1.95	4.15	4.84
	Flexible	3.00	5.83	8.37	1.98	4.19	5.04
Medium	Stiff	3.43	6.03	9.59	2.11	4.23	6.88
	Regular	3.62	6.25	10.20	2.26	4.49	6.95
	Flexible	3.67	6.57	8.72	2.25	4.75	7.14
Fast	Stiff	4.28	6.66	9.74	2.42	4.65	9.55
	Regular	4.40	6.87	10.09	2.54	4.97	9.65
	Flexible	4.79	7.20	10.69	2.49	5.17	10.51

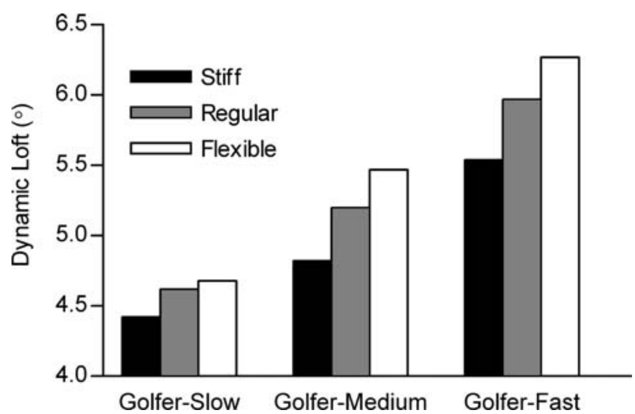

Fig. 5 Dynamic loft of the clubhead at impact for the nine optimized golfer-club models

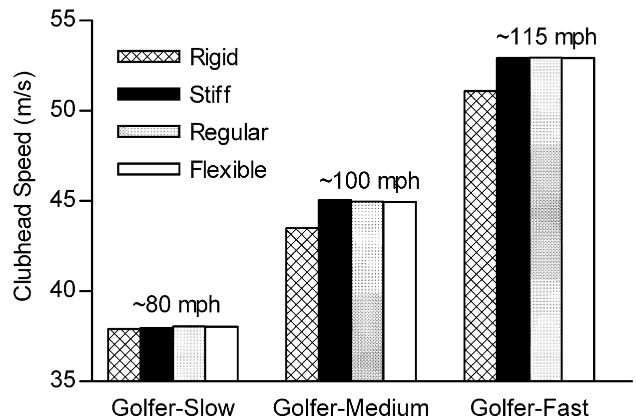

Fig. 6 Clubhead speed at impact for the 12 optimized golfer-club models. The Rigid club condition represents a shaft with infinite stiffness and, as such, does not bend during the downswing

particular level of swing speed occurred within the Golfer-Medium model between Club-Stiff (45.04 m/s) and Club-Flexible (44.96 m/s). The only exception was the lower clubhead speeds attained with Club-Rigid. It should be noted that Club-Rigid is purely a theoretical construct and cannot exist in reality. However, it does provide an indication that, for swing speeds beyond approximately 50 m/s (~115 mph), any non-rigid shaft contributes upwards of 4% to clubhead speed.

The consistent deflections of the clubhead in the lag direction prior to impact (Table 2) and consistent leading positions at impact suggest that the clubhead is 'kicking' forward, which is a potential mechanism that could increase clubhead speed. For all optimized simulations, kick velocity peaked nearly simultaneously with impact (Fig. 7). This suggests that kick velocity played an import role in the overall maximization of clubhead speed. For example, the Golfer-Medium\Club-Regular simulation demonstrated that kick velocity contributed approximately 7 m/s to the final clubhead speed (Table 2).

When the clubhead speed results (Fig. 6) are compared to the kick velocity results (Table 2) there appears to

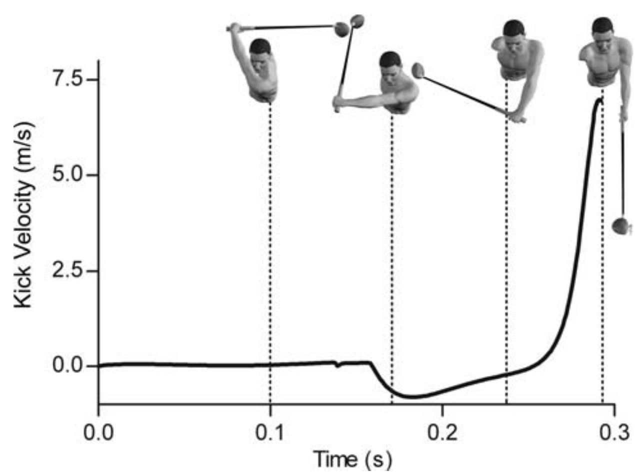

Fig. 7 Kick velocity during the optimized execution of Golfer-Medium with Club-Regular

be an incongruence in the findings. For example, Golfer-Medium attained 1.5 m/s more clubhead speed with Club-Stiff compared to Club-Rigid (Fig. 6). However, the

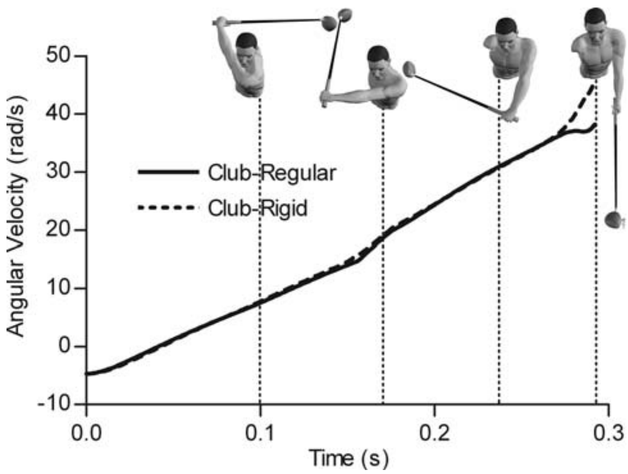

Fig. 8 Angular velocity of the most proximal club segment, *Club_Proximal*, for the optimized simulations of Golfer-Medium with Club-Stiff and Golfer-Medium with Club-Rigid

Golfer-Medium\Club-Stiff combination generated a kick velocity of 6.88 m/s at impact (Table 2). One finding suggests shaft flexibility contributed 1.5 m/s to the clubhead, while the other suggests the contribution was 6.88 m/s. Since the golfer model dynamically interacts with the club model, a rigid club will result in different golfer kinematics compared to a non-rigid club. The angular velocities of the most proximal club segment, about an axis perpendicular to the swing plane, for Golfer-Medium with Club-Stiff and Golfer-Medium with Club-Rigid support this view (Fig. 8). At impact, the most proximal club segment of Club-Rigid had a higher angular velocity (+8.95 rad/s) than that of Club-Stiff. This finding will be explored further in the following section.

4 Discussion

The optimized simulations showed that, for swing speeds beyond approximately 50 m/s ($\sim$115 mph), any non-rigid shaft can contribute up to 4% of the total clubhead speed generated during the downswing. However, our results suggest that matching the stiffness of a golf club shaft to a particular golfer will not increase clubhead speed sufficiently to have any meaningful effect on performance. The difference in clubhead speeds, across levels of shaft stiffness, did not exceed 0.1 m/s for any golfer model. Previous experimental studies have shown that shaft flexibility can increase clubhead speed via the contribution from kick velocity [2, 14]. Even in our study, kick velocities as high as 10.5 m/s at impact were predicted. However, as will be explained, kick velocity can be misleading. During the last third of the downswing, when the clubhead was deflected into a lagging position, the rotational springs generated restoring torques (see Eq. 1). Near impact, the dynamic forces permitted the shaft to recoil from its lagging position into a leading position. This process increased clubhead speed relative to the most proximal club segment. Yet, it also served to simultaneously impede the absolute angular velocity of the most proximal club segment. Particular emphasis is placed on the most proximal club segment, *Club_Proximal*, since it includes the lead hand of the golfer model. Therefore, a reduction in the speed of *Club_Proximal* constitutes a decreased velocity of the model's hand. A major portion of clubhead speed can be attributed to the speed of the model's hand. It is possible that this phenomenon reflects a limitation of our particular model to sufficiently resist the recoil of the shaft. However, for Golfer-Medium, the longitudinal rotation of the arm exceeded 50 rad/s during the recoil of the shaft from its lagged position. As with our model, it would be difficult for a live golfer to generate resistive muscular force at such high angular velocities.

In a similar vein, a golfer does not have the ability to produce constant levels of acceleration during the downswing. This statement is supported by the experimental force/torque measurements reported by previous researchers [5, 15–19] as well as the muscular torque outputs from golfer simulations [20, 21]. The non-constant acceleration profiles previously reported are consistent with the accepted theories regarding the activation rate and force–velocity properties of skeletal muscle [22]. These findings have important implications when considering the potential contribution from kick velocity. Previously, researchers have used golfer models with fixed levels of acceleration during the downswing in their attempt to study shaft flexibility [14, 23]. This is not a reasonable assumption since the golfer model must be able to interact with the dynamic properties of the club. If fixed functions of acceleration were used in this study in place of the muscle torque generators, then the importance of shaft flexibility in contributing to clubhead speed would likely have been greatly over estimated. It is also likely that golfer robots may suffer from this same limitation; namely, the inability to dynamically interact with the properties of a golf club in the same way as a live golfer.

The loft of the clubhead, relative to the ball, at impact will influence both the launch angle and spin rate of the ball. Results from our computer simulations demonstrated that clubhead loft can change by as much as 0.7° depending on shaft stiffness for a golfer with a clubhead speed of approximately 45 m/s ($\sim$101 mph) (Fig. 5). For example, the Golfer-Medium\Club-Stiff simulation resulted in 4.8° of dynamic loft at impact, while the Golfer-Medium\Club-Flexible simulation resulted in 5.5° of dynamic loft. The results from an optimization study conducted by Winfield and Tan [24] suggest that a loft change of this magnitude would be enough to have a meaningful influence on driving distance. However, a golfer must swing consistently to take advantage of these changes.

Kick velocity itself is an interesting phenomenon. According to Butler and Winfield [2], kick velocity is greatest when the shaft is straight at impact because the kinetic energy is maximized. This statement is in agreement with the characteristics of an oscillating spring system and is supported by other researchers [4]. However, our optimized simulations revealed that kick velocity peaked after the clubhead had passed a neutral shaft position. Computer simulation research into the mechanism behind this phenomenon is a potential area of future work.

5 Conclusions

Computer simulation techniques were used to optimize the swings of three golfer models (representing a range of swing speeds) to three drivers (representing a range of shaft stiffness). There was no evidence to support the premise that matching the stiffness properties of the shaft with the golfer would improve clubhead speed. In accordance with previous experimental studies, all optimized swings resulted in the clubhead being deflected in the toe-down and lead directions at impact [2, 12]. Based on our simulations, two generalizations can be suggested for golf swings with similar patterns of force and torque application to the club. (1) As swing speed increases so does the magnitude of shaft deflection at impact. (2) As shaft stiffness decreases the magnitude of shaft deflection at impact increases. The results also indicate that shaft stiffness has a meaningful effect on the effective loft of the clubhead at impact, which would influence the spin rate and launch angle of the golf ball.

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Conflict of interest statement The authors declare that they have no conflict of interest.

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